Homework Assignment 3

1. You arrive to a bus station and wait for your bus to ride. Let us suppose that buses circulate – according to their time-table – at the same intervals in both directions. (This is a typical case for a longer time period of the day in practice as well.) However, due to traffic situations, buses arrive either somewhat sooner or somewhat later than the time-table predicts. You often experience that there is more than one bus going to the opposite direction at the stop where you are waiting before you get your bus. (To be sure; another person waiting for a bus in the opposite direction, in average, experiences the same thing.) Explain this fact in probabilistic terms, based on the so-called waiting-time paradox.

2. Saccharose is decomposing in an acidic solution according to the following equation

   $$\text{Saccharose} + H_2O \rightarrow \text{glucose} + \text{fructose}$$

   In dilute solutions this reaction is pseudo-first order; *i. e.* the reaction is a Poissonian process. Accordingly:

   $$N(t) = N(0)e^{-kt}$$

   where $N(t)$ is the number of saccharose molecules at time $t > 0$, while $N(0)$ is the number of saccharose molecules at time $t = 0$ (the start of the reaction). k is the pseudo-first order rate coefficient. From this it follows that $P(t < T \leq t + dt) = \frac{dN(t)}{N(0)} = ke^{-kt}dt$ for one molecule of saccharose.

   Thus, the p.d.f. of the lifetime of a molecule is $\mathrm{f}(t) = ke^{-kt}$.

   Calculate the expected lifetime of a saccharose molecule in a dilute acidic solution, where the pseudo-first order rate coefficient is $k = 3.46 \times 10^{-4}\ \mathrm{s}^{-1}$.