Assignment 5a: Estimation using the method of moments

We draw a sample of n elements from a continuous uniform distribution of the random variable X. Elements are distributed uniformly between the values a and b, where $a < b$.

Calculate the estimates of a and b from the sample, using the method of moments.

Hint: use the probability distribution function $\frac{1}{b-a}$ to calculate the expectation of X and X^2, and equate them to $\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$ and $\overline{x^2} = \sum_{i=1}^{n} \frac{x_i^2}{n}$, respectively. Solve the resulting equations for a and b. (Make use of the relevant identities factorising differences of powers of b and a.)

Assignment 5b: Estimation of the parameter p of a binomial distribution

We draw a sample of the number of girls in 12 families, all of them having four children. The 12 sample elements are the following:

3, 2, 2, 0, 1, 3, 1 , 1, 1, 2, 3, 1

Calculate the estimate of the probability that a girl is born into a family.
(Comment on the result in view of the well-known global population statistics.)