0.1 Heat and work

1. A particular gas is enclosed in a cylinder with a moveable piston. It is observed that if the walls are adiabatic, a quasi-static increase in the volume results in a decrease in pressure according to the equation:

$$P^3V^5 = \text{constant} \qquad (\text{for} \quad Q = 0)$$

   (a) Find the quasi-static work done on the system and the net heat transfer to the system in each of the three processes (ADB, ACB, and the direct linear process AB) as shown in the figure [see blackboard].

   (b) A small paddle is installed inside the system and is driven by an external motor (by means of a magnetic coupling through the cylinder wall). The motor exerts a torque, driving the paddle at an angular velocity ω, and the pressure of the gas (at constant volume) is observed to increase at a rate given by

$$\frac{dP}{dt} = \frac{2}{3}\frac{\omega}{V} \cdot \text{torque}$$

   Show that the energy difference of any two states of equal volumes can be determined by this process. In particular, evaluate $U_C - U_A$ and $U_D - U_B$. Explain why this process can proceed only in one direction (vertically upward rather than downward in the $P - V$ plot).

   (c) Show that *any* two states (any two points in the $P-V$ plane) can be connected by a combination of the processes in (a) and (b). In particular, evaluate $U_D - U_A$.

   (d) Calculate the work W_{AD} in the process A $\longrightarrow$ D. Calculate the heat transfer Q_{AD}. Repeat for D $\longrightarrow$ B, and for C $\longrightarrow$ A. Are these results consistent with those of (a)?

2. For a particular gaseous system it has been determined that the energy is given by

$$U = 2.5PV + \text{constant}$$

   (a) Calculate Q and W along the parabola $P = 10^5 + 10^9 \cdot (V - 0.02)^2$ between the points A(0.2 MPa, 0.01 m^3) and B(0.2 MPa, 0.03 m^3).

   (b) Find the equation of the adiabats in the $P - V$ plane (i.e. find the form of the curves $P = P(V)$ such that đ$Q = 0$ along the curves).